

Detecting A Gerrymander: Sampling the Space of Possible NC Congressional Maps

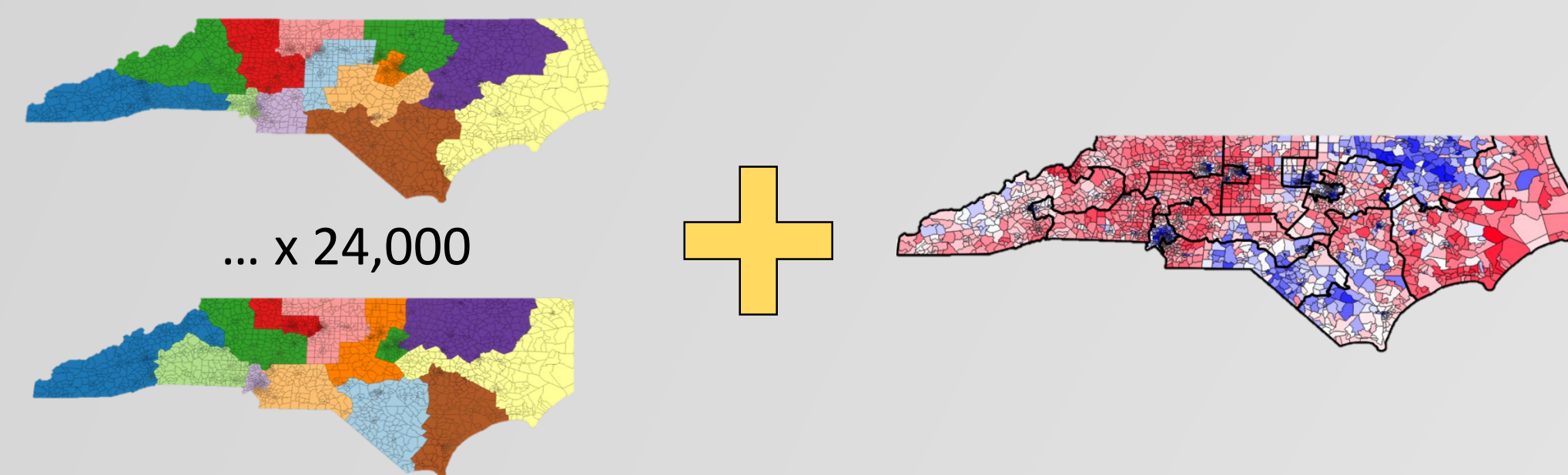
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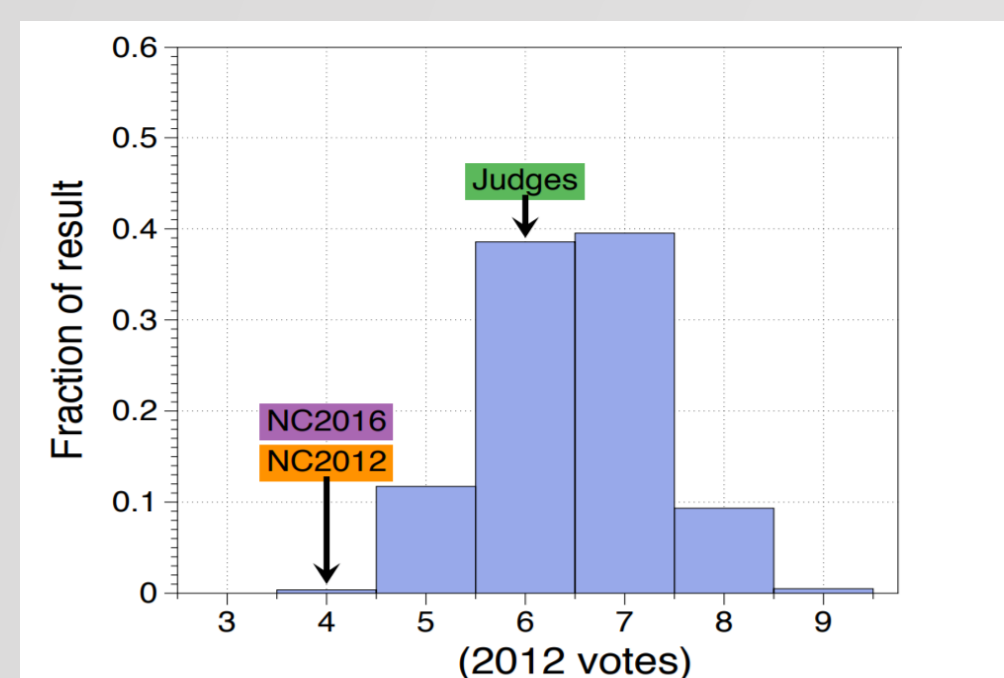
Bass Connections in
Information, Society & Culture

How to Catch A Gerrymander

1. Create A Bunch of Maps
2. Add Real Voting Data



3. Check for outliers on the distribution of election outcomes



Build an Ensemble of Maps

Why not make every possible map?

Over 13^{300} possibilities! Have to take a sample.

Can you take a random sample?

A vast majority of possible maps are illegal! Only want a sample of possibly compliant maps.

How do you hunt for compliant maps?

1. Build a score function as a heuristic for compliance:

α_1 = Contiguity
 α_2 = Compactness
 α_3 = Population Equality
 α_4 = County Splitting
 α_5 = Voting Rights Act

$$\text{Map Score} = \sum_i \alpha_i \omega_i$$

* Weight ω_i corresponds to α_i

2. Assign a probability of generating each map based on this score (β is a tuning parameter)

$$P(\text{map}) \propto e^{-\beta(\text{Map Score})}$$

3. Take steps from map to map by swapping precincts with these probabilities as your guide.

Sampling the Space of Possible Maps

Challenges:

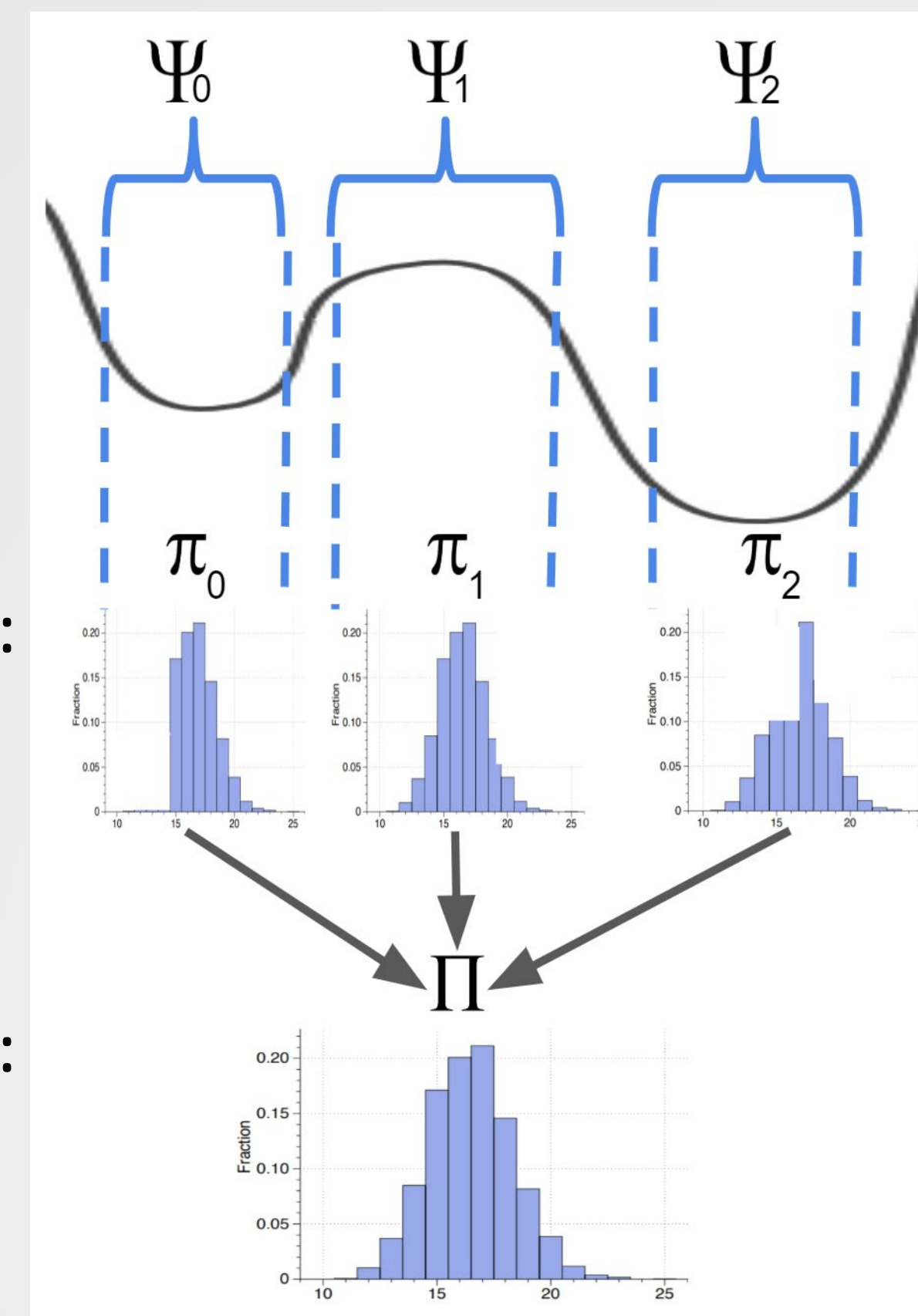
- The space is huge, high dimensional, and hard to navigate.
- It's easy to get stuck in a region of space and miss out on other good maps.
- Must balance *exploration* and *exploitation* while taking millions of steps the space.

Stratified Sampling

1. Define strata: Boundaries that prevent you from stepping too far away from your start point.

2. Create Local Distributions of Election Outcomes: Measure steps attempted from one strata to another.

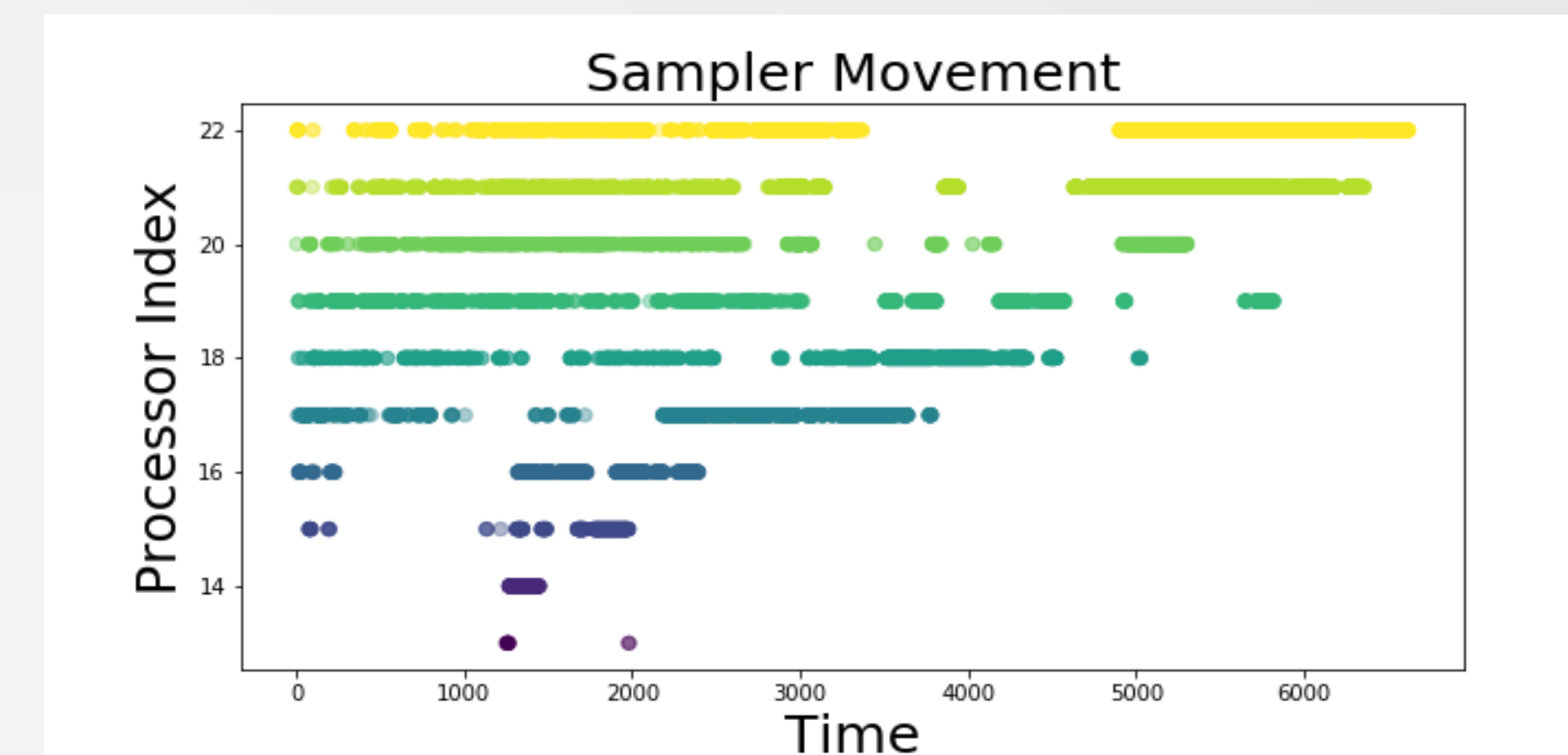
3. Build a Global Distribution of Election Outcomes: Merge all the local distributions to get a complete election outcome distribution.



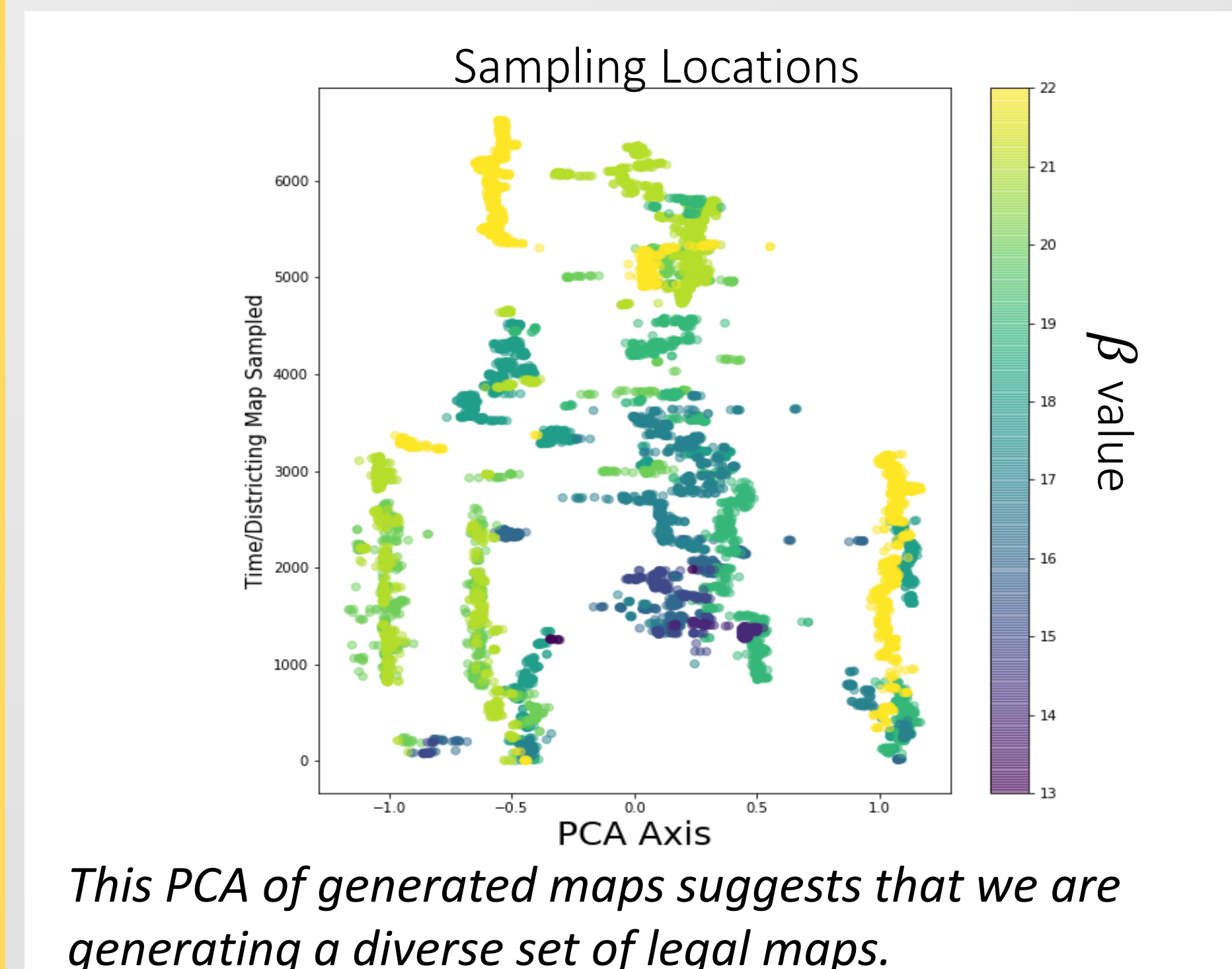
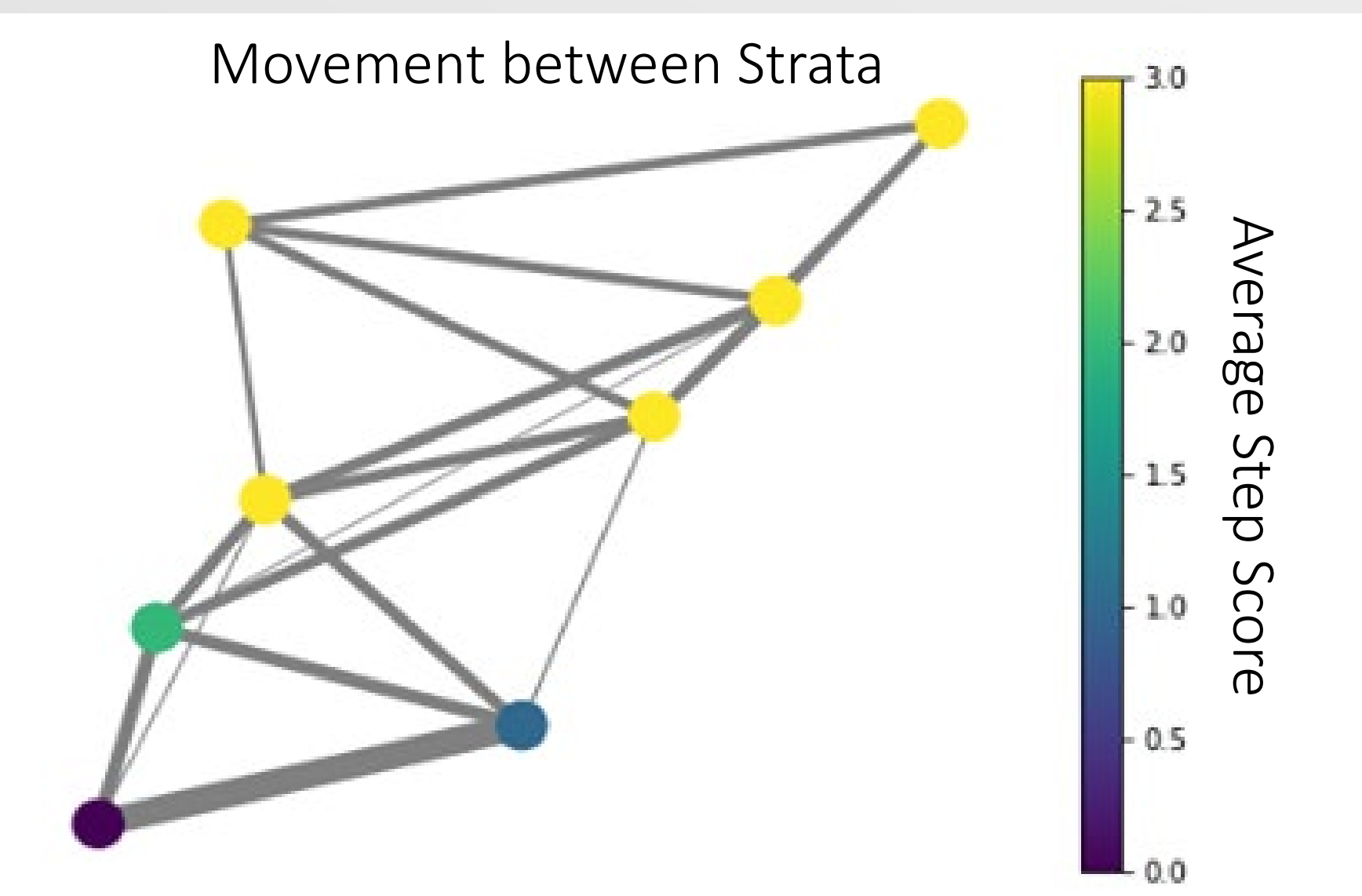
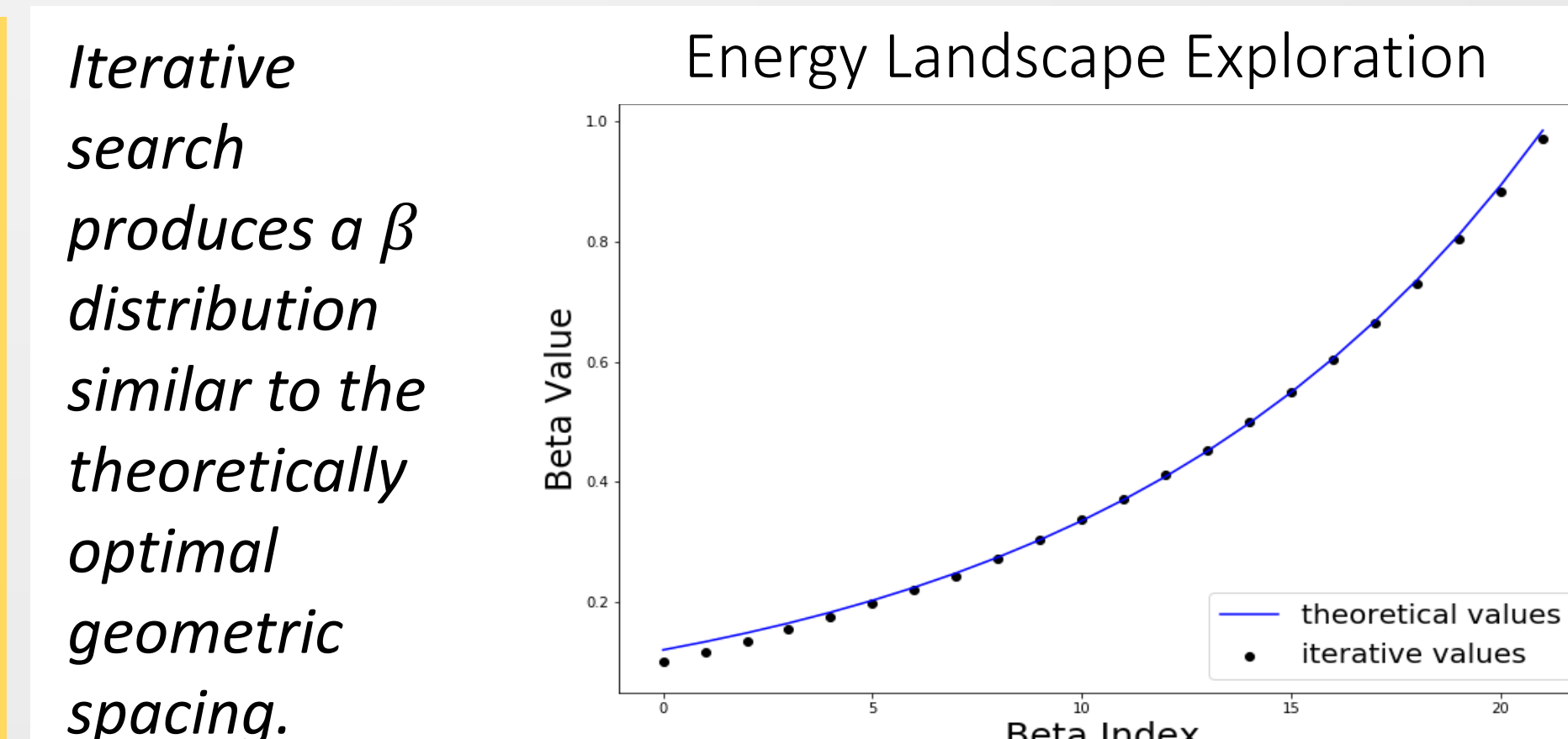
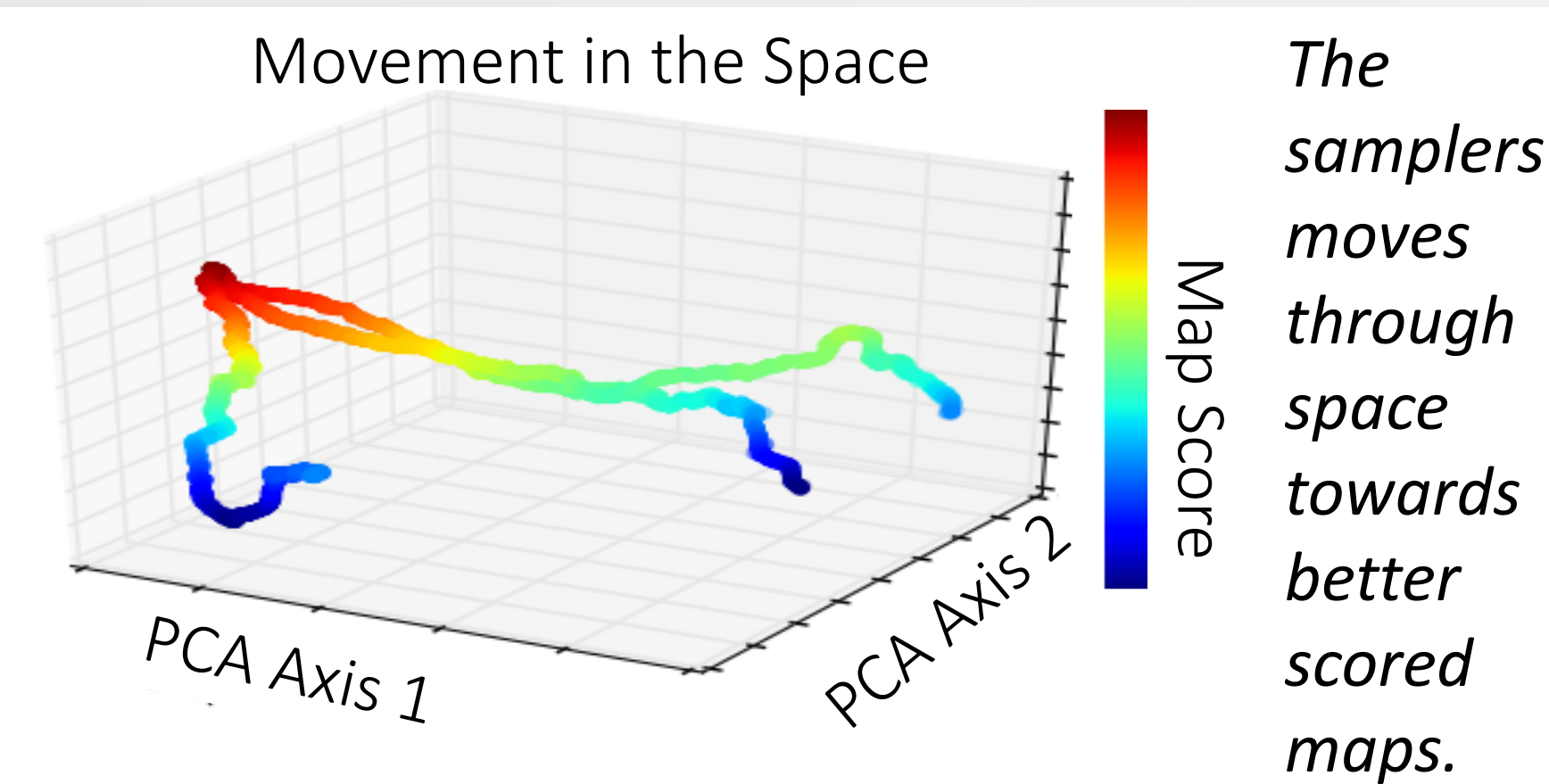
Parallel Tempering Sampling

1. Iteratively explore energy landscape to find an ideal set of β values

2. Run N samplers in parallel, each with different β values, and periodically exchange positions in the energy landscape

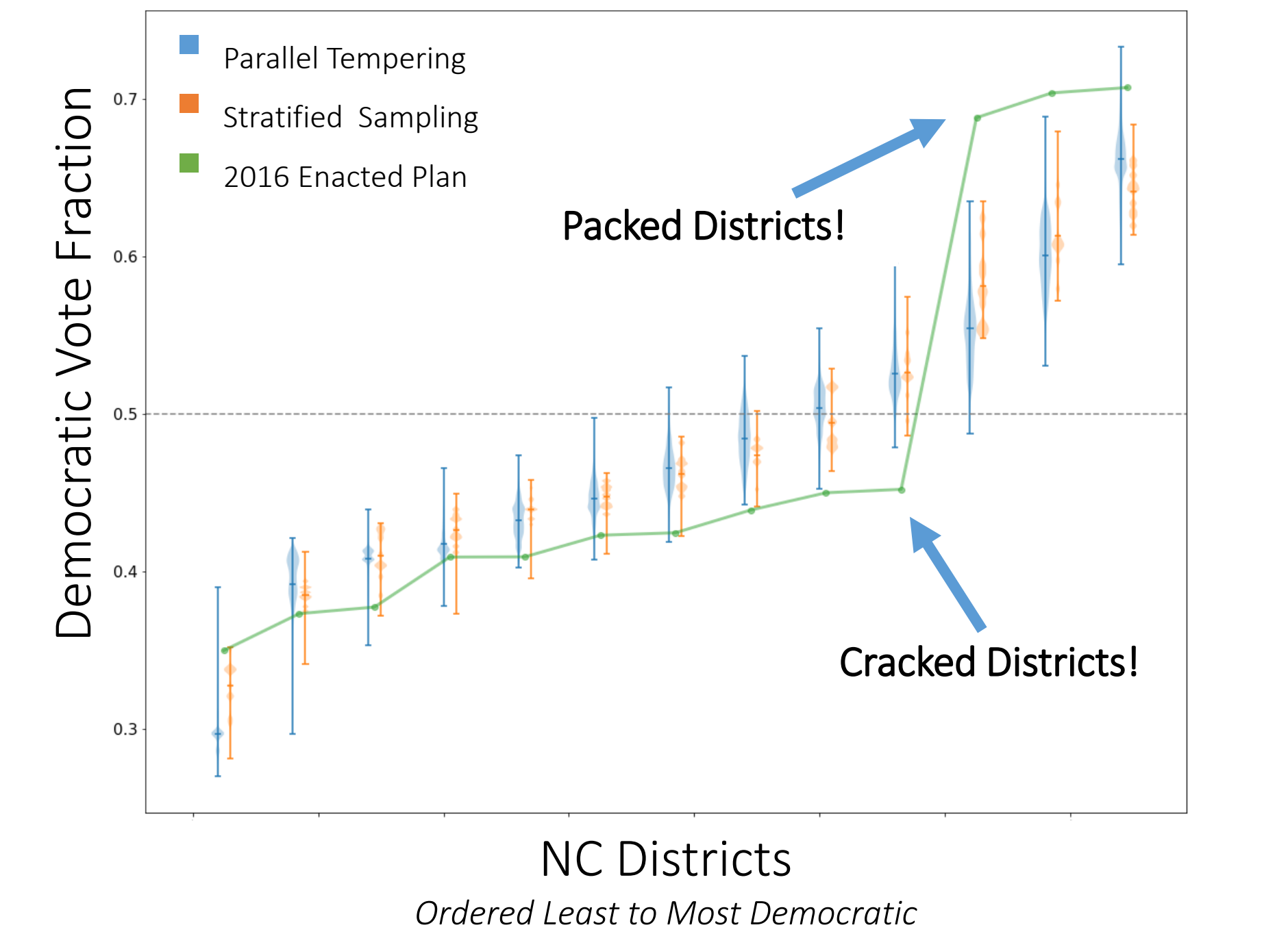


How do we know we sampled well?



Detecting A Gerrymander

Sample Outcomes: The Signature of Gerrymandering



This graph highlights packing and cracking behavior characteristic of gerrymandering while still capturing the natural partisan landscape of the state. Evidence very similar to this was presented to the **Supreme Court in Rucho v. Common Cause**

Takeaways:

1. Consistency across both sampling methods suggests robustness of results.
2. We hypothesize that this S-shaped curve is diagnostic of partisan gerrymandering.
3. 2016 Enacted map is an outlier across districts **Suggesting Partisan Gerrymandering in NC**

Comparing Methods

	Stratified Sampling	Parallel Tempering
Exploration	Can explore many regions at simultaneously.	Can traverse to new regions faster.
Exploitation	By enforcing boundaries guarantee thorough sampling.	By swapping β values can focus on regions with good maps.

Combining Methods Parallel-Stratified Sampling

Take the best parts of both methods to make a novel approach:

1. Assign each stratum its own β value
2. Periodically swap β values between strata
3. Vary the size and nature of strata radii